## SOLUTIONS OF EXERCISES OF WEEK THREE

**Exercise 1.** Suppose that there are two sentences $P$ and $Q$ such that

$$P \Leftrightarrow (P \Rightarrow Q).$$

Show that $Q$ is true.

*Solution.* We argue by contradiction. Suppose that $Q$ is false. We that $P$ is neither true or false and obtain a contradiction.

If $P$ is true, then

$$P \Rightarrow Q$$

is false. Then $P$ is false and we obtain a contradiction.

Now, suppose that $P$ is false. Then $P \Rightarrow Q$ is true. Then $P$ is true and we obtain another contradiction.

Then $Q$ is true. □

**Exercise 2.** Translate into the formal language the following sentence

*"For every y there exists a unique x such that f(x) = y"*

*Solution.* A way to translate the sentence in formal language is

$$\forall y \exists x \Big( (f(x) = y) \wedge \big( \forall z (f(z) = y \Rightarrow z = x) \big) \Big)$$

□

**Exercise 3.** In the following table

| $\in$ | $A$ | $B$ | $C$ | $D$ |
|---|---|---|---|---|
| $A$ | 0 | 1 | 0 | 1 |
| $B$ | 1 | 0 | 0 | 1 |
| $C$ | 0 | 1 | 0 | 0 |
| $D$ | 0 | 0 | 0 | 0 |

Find the elements and the proper classes. State whether the following classes exist. In the affermative case, find the classes they correspond to (for example, the empty class exists and $\emptyset = C$).

1. the complement class $A'$
2. $A \cap B$
3. $B \cup C$
4. the universal class $\mathscr{U}$.

Is the Class Construction Axiom satisfied?

*Date*: 2013, October 1.

*Solution.* The elements are

$$A, B, C;$$

there is only a proper class

$$D.$$

1. Since $A = \{B\}$, the complement $A' = \{A, C\} = B$

2. $A \cap B = \emptyset = C$

3. $B \cup C = B$

4. the universal class does not exists: no class contains all the elements $A, B, C$

5. the Class Construction Axiom does not hold because the Universal Class does not exists.

$\square$