# EXERCISES OF WEEK FOURTEEN

**Exercise 1.** State whether the class $P_i$ is a partition of $A_i$.

(1) $$P_1 = \{\{a\}, \{b, c\}\}, \quad A_1 = \{a, b, c, d\}$$

(2) $$P_2 = \{\{x, y\}, \{y, z\}\}, \quad A_2 = \{x, y, z\}$$

(3) $$P_3 = \{\{x, y\}, \{z\}, \emptyset\}, \quad A_3 = \{x, y, z\}$$

*Proof.* We suppose that $a, b, c, d$ are all different from each other. Then $P_1$ is not a partition of $A_1$ because

$$\cup P_1 \neq A_1.$$

$P_2$ is not a partition of $A_2$, because

$$\{x, y\} \cap \{y, z\} \neq \emptyset.$$

$P_3$ is not a partition of $A_3$ because $\emptyset \in P_3$. □

**Exercise 2.** Let $(A, \leq)$ be a partially ordered class: show that

(a) $S_a \cap S_b$ is an initial segment, **if A is a fully-ordered class**
(b) there exists a p.o.c $(A, \leq)$ such that $S_a \cup S_b$ is not an initial segment
(c) if $(A, \leq)$ is a fully ordered class, then $S_a \cup S_b$ is an initial segment

*Proof.*

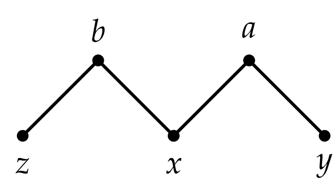
(a) Since $A$ is a fully-ordered class, either $a \leq b$ or $b < a$. If $a \leq b$, then $S_a \subseteq S_b$, hence

$$S_a \cap S_b = S_a,$$

so $S_a \cap S_b$ is an initial segment. If $b < a$, then $S_b \subseteq S_a$ and

$$S_a \cap S_b = S_b$$

so $S_a \cap S_b$ is an initial segment.
Notice that, if $A$ is not a fully-ordered class, then $S_a \cap S_b$ might not be an initial segment:




Clearly, $S_a = \{x, y\}$ while $S_b = \{z, x\}$, while $S_a \cap S_b = \{x\}$ which is not an initial segment.

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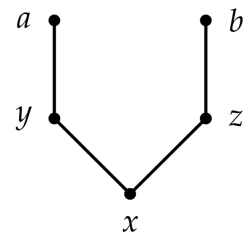
*Date*: 2013, December 4.

(b) the following example explains why $S_a \cup S_b$ might not be an initial segment, if $A$ is not a fully-ordered class:




$S_a = \{x, y\}$, $S_b = \{x, z\}$ and $S_a \cup S_b = \{x, y, z\}$ which is not an initial segment.

(c) if $A$ is a fully-ordered class, then either $a \leq b$ or $b < a$. If $a \leq b$, then $S_a \subseteq S_b$ and $S_a \cup S_b = S_b$. If $b < a$, then $S_b \subseteq S_a$ and $S_a \cup S_b = S_a$. In either cases, the union is an initial segment.

□