SOLUTIONS OF THE EXERCISES OF THE BOOK - WEEK TEN

Exercises of page 155.

Exercise 5. Is there a potential for $\boldsymbol{F}(x,y) = (y^2 + 3x^2, 2xy)$? If so, find one.

Solution. Since $\nabla \times \boldsymbol{F} = 0$ and $\mathbb{R}^2$ is simply connected, there exists a potential;

$$\partial_x g = y^2 + 3x^2 \Rightarrow g(x,y) = y^2 x + x^3 + c(y).$$

Then

$$\partial_y g = 2xy + c'(y) = 2xy \Rightarrow c'(y) = 0 \Rightarrow c(y) \equiv c.$$

Then, a potential is

$$g(x,y) = y^2 x + x^3$$

□

Exercise 6. Is there a potential for

$$\boldsymbol{F}(x,y) = (x^3 \cos(xy) + 2x \sin(xy), x^2 y \cos(xy))?$$

if so, find one

Solution. We have

$$\nabla \times \boldsymbol{F} = 2xy\cos(xy) - x^2y^2 \sin(xy) + x^4 \sin(xy) - 2x^2 \cos(xy) \neq 0.$$

So, the vector field is not conservative. □

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