EXERCISES OF WEEK TWELVE

Exercise 1. Given open sets $\Omega, \Omega' \subseteq \mathbb{R}^2$ and C^1 function
$$\psi\colon \Omega \to \mathbb{R}^3, \quad \varphi\colon \Omega' \to \Omega$$
we can define the composition
$$g(u,v) := \psi(\varphi(u,v)).$$
Using the chain rule (check Theorem 4.11, page 166 of the book of M. Corral), show that
$$\partial_u g(u,v) \times \partial_v g(u,v) = \big(\partial_u \varphi(u,v) \times \partial_u \varphi(u,v)\big)(\partial_x \psi(x,y) \times \partial_y \psi(x,y)).$$

Exercise 2. The following function
$$\varphi\colon (0,1) \times (0,2\pi) \to \mathbb{R}^2, \quad \varphi(\rho,\vartheta) = (\rho\cos\vartheta, \rho\sin\vartheta)$$
is $C^1(\Omega;\mathbb{R}^2)$, where
$$\Omega = (0,1) \times (0,2\pi).$$

a) Show that
$$\varphi(\Omega) = U$$
where
$$B(O,1) - \{(x,0) \mid 0 \leq x < 1\};$$
b) show that $\varphi\colon \Omega \to U$ is a variable change, that is, φ is injective and
$$\partial_x \varphi \times \partial_y \varphi(x,y) \neq 0$$
for every $(x,y) \in \Omega$.

Exercise 3. We define the parametric surface
$$\psi\colon B(O,1) \to \mathbb{R}^3, \quad \psi(x,y) = (x, y, x^2 - y^2)$$

a) is ψ injective?
b) evaluate $\psi_x \times \psi_y$
c) what is the area of ψ (the variable change of the second exercise can be helpful)?

Date: 2013, November 21.