

## EXERCISES OF WEEK TEN

**Exercise 1.** Given the function

$$g: \mathbb{R} \rightarrow (-\pi/2, \pi/2), \quad g(s) = \arctan(s)$$

show that

$$g(1/s) = \frac{\pi}{2} - g(s)$$

for every  $s > 0$  and

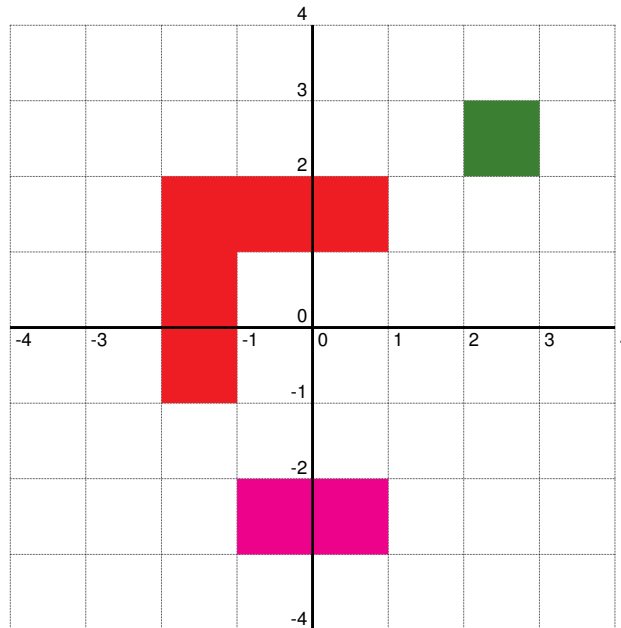
$$g(1/s) = -\frac{\pi}{2} - g(s)$$

for every  $s < 0$ .

**Exercise 2.** Find the potential of the vector field

$$\mathbf{X} = \frac{1}{2\pi} \left( -\frac{y}{x^2 + y^2}, \frac{x}{x^2 + y^2} \right)$$

on the following regions:



$$\Omega_1 := \{(x, y) \mid 2 < x < 3, 2 < y < 3\}$$

$$\Omega_2 := \{(x, y) \mid -1 < x < 1, -3 < y < -2\}$$

$$\Omega_3 := \{(x, y) \mid -2 < x < 1, 1 < y < 2\} \cup \{(x, y) \mid -2 < x < 2, -3 < y < -2\}$$

**Exercise 3.** An ellipse of axes  $a$  and  $b$  can be parametrized with the curve

$$\alpha: [0, 1] \rightarrow \mathbb{R}^2, \quad \alpha(t) = (a \cos 2\pi t, b \sin 2\pi t)$$

Using the Green's theorem, find the area of the ellipse.