EXERCISES OF WEEK FIVE

Exercise 1. Given three lines $\ell_1 := \ell(P, v), \ell_2 := \ell(Q, w)$ and $\ell_3 := \ell(R, z)$ such that

$$\ell_i \neq \ell_j \text{ if } i \neq j$$

find necessary and sufficient conditions in terms of $P, Q, R \in \mathbb{R}^3$ and $v, w, z \in E^3$ in order to have

$$\ell_1 \cap \ell_2 \cap \ell_3 \neq \emptyset.$$

Exercise 2. We define the two-variable function

$$g(x, y) = \begin{cases} \frac{x^2 y}{x^2+y^2} & \text{if } x^2 + y^2 \neq 0 \\ 0 & x = y = 0 \end{cases}$$

State whether[1]

1. g is bounded on $B((0, 0), 1)$
2. g is continuous at the point $O(0, 0)$
3. the partial derivatives $\partial_x g(O)$ and $\partial_y g(O)$ exist
4. the partial derivatives $\partial_x g$ and $\partial_y g$ are bounded on $B((0, 0), 1)$
5. g is differentiable at $O(0, 0)$
6. g is smooth on $B((0, 0), 1)$

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[1]The inequality $2xy \leq x^2 + y^2$ is useful in this exercise