SOLUTIONS OF THE EXERCISES OF WEEK FOUR

Exercise 1. State whether the two lines ℓ and ℓ' (defined below) intersect. If the intersection is non-empty, then find the intersection points

$$\ell := \ell((-1,1,0),(2,1,3)), \quad \ell' := \ell((1,1,0),(0,1,1)).$$

Solution. We set

$$v := (2,1,3), \quad w = (0,1,1), \quad P(-1,1,0), \quad Q(1,1,0).$$

We have

$$v \times w = (-2,2,2) \neq 0, \quad \overrightarrow{PQ} = (2,0,0)$$

and

$$\overrightarrow{PQ} \cdot (v \times w) \neq 0.$$

Then the intersection is empty. □

Exercise 2. Write the normal form of the plane containing the two lines ℓ and ℓ' in the first exercise.

Solution. Since $\overrightarrow{PQ} \cdot (v \times w) \neq 0$ no plane contains both the lines. □

Exercise 3. Find the distance between the point $Q(2,1,3)$ and the plane given in normal form

$$\pi(P(1,0,1),(0,1,2)).$$

Solution. We apply the distance formula

$$\overrightarrow{PQ} = (1,1,2), \quad \overrightarrow{PQ} \cdot (0,1,2) =$$

$$\mathrm{dist}(Q,\pi) = \frac{|\overrightarrow{PQ} \cdot v|}{\|v\|} = \frac{5}{\sqrt{5}} = \sqrt{5}.$$

□

Exercise 4. Find the intersection between the two planes

$$\pi(P(1,0,1),(1,2,0)) \cap \pi(P(2,1,3),(1,0,1)).$$

Solution. We set

$$v = (1,2,0), \quad w = (1,0,1).$$

Then

$$v \times w = (2,-1,-2).$$

In order to find an intersection point, we solve the system

$$\begin{cases} x - 1 + 2y = 0 \\ x - 2 + z - 3 = 0. \end{cases}$$

We set $x = 0$ and obtain

$$y = \frac{1}{2}, \quad z = 5.$$

Then the intersection line is

$$\ell((0,1/2,5),(2,-1,-2)).$$

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$\square$

Exercise 5. Find the distance between the point $P(1,2,2)$ and the line ℓ in the first exercise.

Solution. We apply the distance formula in dimension-two.

$$\overrightarrow{PQ} = -(2,1,2), \quad \overrightarrow{PQ} \times v = -(2,1,2) \times (2,1,3) = (-1,2,0)$$

Then

$$\text{dist}(P,\ell) = \frac{\|\overrightarrow{PQ} \times v\|}{\|v\|} = \frac{\sqrt{5}}{\sqrt{14}}$$

$\square$