A BRIEF HISTORY OF SET THEORY

This is a short (and simplified) history of set theory, divided in a few periods.

1. **The Constructivism.** The 19th century is the century of the Constructivism (in Mathematics). The constructivism asserts that it is necessary to construct or find an object, in order to show that such object exists.

Consequently, sets allowed to exists are only the finite ones. The existence of the set of natural numbers $\mathbb{N}$ was taken for granted because it was believed that it was somewhat part of the real world. One of the most eminent representative was Leopold Kronecker (1823-1891, Professor at University of Berlin).

2. **Georg Cantor (Saint Petersburg, 1845 - Halle 1916).** While solving a problem of uniqueness of Fourier expansion, Georg Cantor introduced new sets (as the set of real numbers and transfinite numbers) and rules of making sets, mainly through the Axiom of Unrestricted Class Comprehension (UCS).

The Axiom of UCS delivers non-constructive methods and represented a challenge to the philosophy of the Constructivism. While he was praised for his results on Fourier series (and became later popular among his successors for all his contributions to the set theory), he was heavily criticized by the most eminent mathematicians of that time, including Leopold Kronecker.

He became established as a professor at the University of Halle after being rejected by the University of Berlin as a ”corrupter of youth” (Kronecker) and having his paper rejected for his ”too much uninhibited use of sets”.

3. **Bertrand Russell (United Kingdom, 1872 - 1970).** In 1901, through the famous example about sets which are not elements of themselves, Bertrand Russell proved that the Axiom of Unrestricted Class Comprehension is contradictory.

At that time the Axiom was considered a common tool, real numbers a strict consequence of it, and the theory of transfinite number a exciting and beautiful part of Mathematics. The failure of the Axiom meant that suddenly they potentially had to relinquish all of them.

4. **Axiomatic approach.** Since a lot of scientists wished to retain the nice consequences of the Axiom UCS, the next challenge was fixing the existing (and failing) set theory. And possibly trying not to drop real numbers, for example.

A lot of mathematicians and philophers agreed that until then sets had not met a completely rigorous approach; most of the knowledge relied on intuition or contradictory axioms (as the UCS) and was sometimes delivered through informal language.

Then several axiomatic approaches to Set Theory were delivered, as the Zermelo-Fraenkel-Choice (ZFC, 1920), the Neumann-Bernays-Gödel (NBG, 1930), Morse-Kelley (MK, 1965). The second is the one we are interested on.

The forerunner set theory, along with its devices and contradictions was referred to as Naive Set Theory.