Es. 2

$$\Gamma = \{(x,y) \in \mathbb{R}^2 : \underbrace{(y^2-1)e^{xy}}_{F(x,y)} = 0\}$$

$$P_0 = (2,1)$$

$$\nabla F(x,y) = \left((y^2-1)ye^{xy},\ e^{xy}\left(2y + x(y^2-1)\right) \right)$$

$$\Longrightarrow \nabla F(P_0) = (0, 2e^2) \neq \underline{0}$$

$$\frac{\partial F}{\partial y}(P_0) \neq 0 \Longrightarrow \exists!\ g : B_\delta(2) \longrightarrow B_{\delta'}(1)$$

$(F \in \mathcal{C}^1)$ Teo. funz. implicita

di classe $\mathcal{C}^1$ tale che $g(2) = 1$ e

$$F(x, g(x)) = 0$$

$$\Longleftrightarrow \left(g^2(x) - 1\right)\cancel{e^{x\,g(x)}} = 0$$

$$\Longleftrightarrow g(x) = 1 \quad \vee \quad \cancel{g(x) = -1}$$

no perché $g(2) = 1$

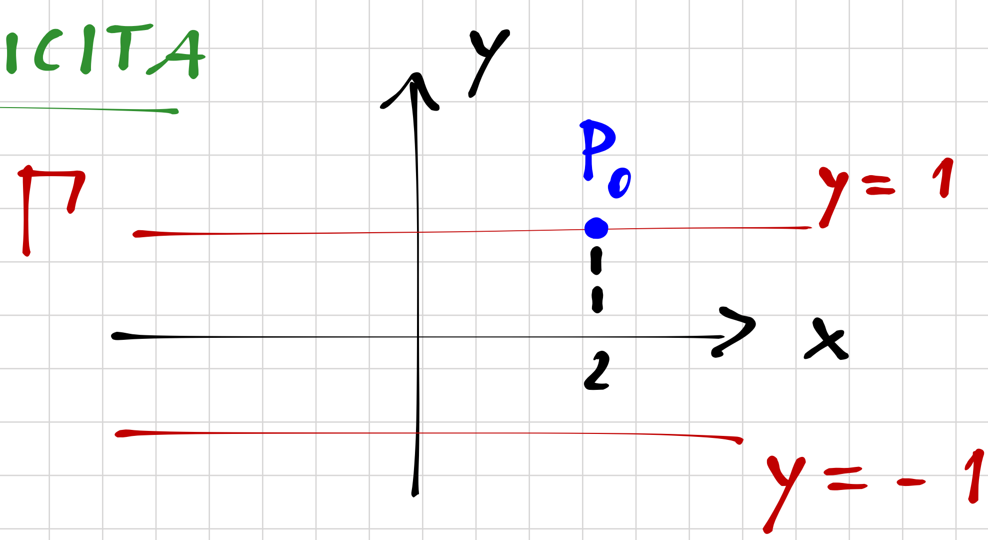
è parametrizz. locale di Γ in P_0

SENZA TEO. FUNZ. IMPLICITA

$$(y^2-1)e^{xy} = 0$$

$$\Updownarrow$$

$$y = 1 \quad \vee \quad y = -1$$



Es. 3

$$\Gamma = \{(x,y)\in\mathbb{R}^2 : \underbrace{x^3+y^2-y(x+x^2)-1}_{F(x,y)}=0\}$$

$$P_0=(1,0)\in\Gamma$$

$$\nabla F(x,y) = \left(3x^2-y(1+2x),\ 2y-x-x^2\right)$$

$$\nabla F(1,0) = (3,-2)\neq \underline{0} \qquad (*)$$

$\dfrac{\partial F}{\partial y}(P_0)\neq 0 \implies$ Γ è parametrizzabile in un intorno di P_0 come grafico di $y=g(x)$

$$x^3+g^2(x)-g(x)(x+x^2)-1=0$$

$$g^2(x)-(x+x^2)\,g(x)+x^3-1=0$$

$$g(x)=\frac{x+x^2\pm\sqrt{(x+x^2)^2-4(x^3-1)}}{2}$$

g deve soddisfare $g(1)=0$, quindi

$$g(x)=\frac{1}{2}\left(x+x^2-\sqrt{(x+x^2)^2-4(x^3-1)}\right)$$

$(*) \implies 3(x-1)-2y=0$ eq. retta tang. a Γ in P_0

Es. 4

$$\Gamma = \{(x,y)\in\mathbb{R}^2 : \underbrace{y^3 + \log(x+y)}_{F(x,y)} = 0\}$$

$$P_0 = (1,0) \in \Gamma$$

$$\nabla F(x,y) = \left(\frac{1}{x+y},\ 3y^2 + \frac{1}{x+y}\right)$$

$$\nabla F(P_0) = (1,1) \neq \underline{0}$$

$\Rightarrow$ Γ ha retta tang. in P_0

EQ. CARTESIANA	EQ. PARAMETRICHE
$1(x-1)+1y=0$ $x+y=1$	$\begin{cases} x = 1-\lambda \\ y = \lambda \end{cases}, \quad \lambda\in\mathbb{R}$

$$\frac{\partial F}{\partial x}(P_0) \neq 0 \implies \exists!\, h : B_\delta(0) \to B_{\delta'}(1)$$

derivabile, $h(0) = 1$,

$F(h(y), y) = 0$

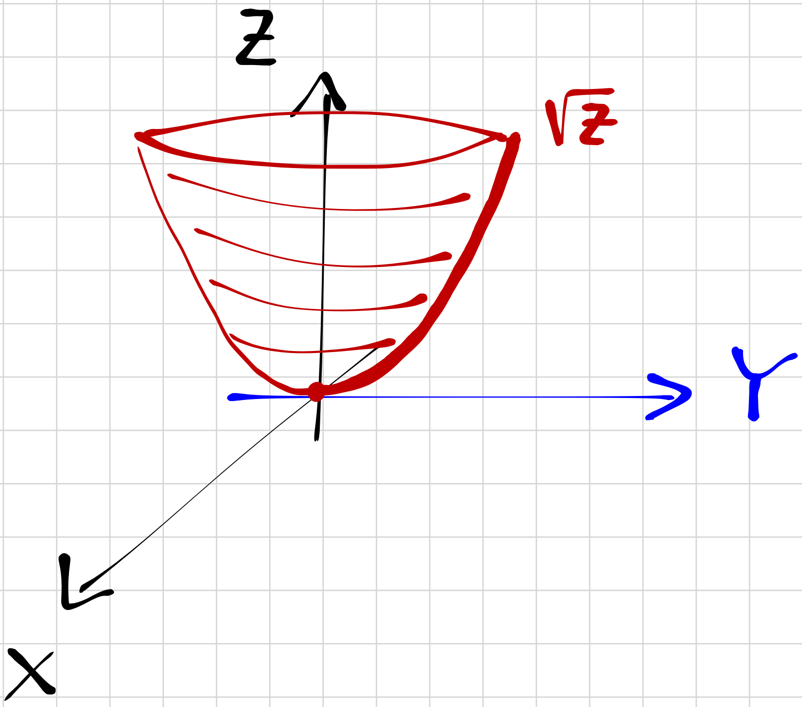
$$y^3 + \log(h(y)+y) = 0$$

$h(y) = e^{-y^3} - y$ parametrizz. locale di Γ

Es. 5

$$\Sigma = \{ x^2 + (y-2)^2 - z = 0 \}, \quad P_0 = (1,2,1)$$

$$x^2 + (\underbrace{y-2}_{Y})^2 = z = (\sqrt{z})^2$$



$$\begin{cases} x = \sqrt{t}\cos\theta \\ Y = \sqrt{t}\sin\theta \\ z = t \end{cases}$$

con $t \geq 0$ e $\theta \in [0, 2\pi)$

$\Rightarrow$ Σ si parametrizza globalmente con $\sigma : [0,+\infty) \times [0,2\pi) \longrightarrow \mathbb{R}^3$

$$\sigma(t,\theta) = \begin{pmatrix} \sqrt{t}\cos\theta \\ \sqrt{t}\sin\theta + 2 \\ t \end{pmatrix}$$

$\Rightarrow$ matrice Jacobiana di σ

$$J\sigma(t,\theta) = \begin{pmatrix} \frac{1}{2\sqrt{t}}\cos\theta & -\sqrt{t}\sin\theta \\ \frac{1}{2\sqrt{t}}\sin\theta & \sqrt{t}\cos\theta \\ 1 & 0 \end{pmatrix}$$

Cerchiamo $(t_0, \vartheta_0) \in D$ tale che $\sigma(t_0, \vartheta_0) = P_0 = (1,2,1)$:

$$\begin{cases} \sqrt{t_0}\cos\theta_0 = 1 \\ \sqrt{t_0}\sin\theta_0 + \cancel{2} = \cancel{2} \\ t = 1 \end{cases} \iff \begin{cases} t_0 = 1 \\ \cos\theta_0 = 1 \\ \sin\theta_0 = 0 \end{cases}$$

$$\iff (t_0, \vartheta_0) = (1,0)$$

$$J\sigma(1,0) = \begin{pmatrix} 1/2 & 0 \\ 0 & 1 \\ 1 & 0 \end{pmatrix}$$ ha rango 2 $\Rightarrow \exists$ piano tang. a Σ in P_0

$$\vec{n}(1,0) = \frac{\partial\sigma}{\partial t}(1,0) \times \frac{\partial\sigma}{\partial\vartheta}(1,0) = \begin{pmatrix} -1 \\ 0 \\ 1/2 \end{pmatrix}$$

$$\Rightarrow -1\cdot(x-1) + \frac{1}{2}(z-1) = 0$$

$2x - z = 1$ eq. cartesiana del piano tang. a Σ in P_0

Es. 9

$$\Sigma = \{ \underbrace{xy^2 - y + \log(1+x^2z^2)}_{F(x,y,z)} = 1 \}$$

$$P_0 = (0, -1, 1) \in \Sigma$$

$$\nabla F(x,y,z) = \left(y^2 + \frac{2xz^2}{1+x^2z^2}, 2xy - 1, \frac{2x^2z}{1+x^2z^2} \right)$$

$$\nabla F(P_0) = (1, -1, 0) \neq \underline{0}$$

$\Rightarrow$ $x - y = 1$ piano tang. a Σ in P_0

$\frac{\partial F}{\partial y}(P_0) \neq \underline{0} \Rightarrow$ Σ localmente graf. di una funz. $y = g(x,z)$

$$x g^2(x,z) - g(x,z) + \log(1+x^2z^2) - 1 = 0$$

$$g(x,z) = \frac{1 \pm \sqrt{1 - 4x(\log(1+x^2z^2) - 1)}}{2x}, \quad x \neq 0$$

$$g(0,1) = -1 \Rightarrow g(x,z) = \frac{1 - \sqrt{1 - 4x(\log(1+x^2z^2) - 1)}}{2x}$$

se $x \neq 0$

g deve essere continua $\Longrightarrow \forall z \quad g(0,z) = \lim_{x \to 0} g(x,z) = -1$

ESERCIZIO

Es. 11

$$\Sigma = \{ \underbrace{\log(1+x^2) + xy}_{F(x,y,z)} = 0 \} \subseteq \mathbb{R}^3$$

$$P_0 = (0, 2, 1) \in \Sigma$$

$$\nabla F(x,y,z) = \left(\frac{2x}{1+x^2} + y, \ x, \ 0 \right)$$

$$\nabla F(P_0) = (2, 0, 0) \neq \underline{0}$$

$$\Longrightarrow \cancel{2}(x-0) = 0$$

$x = 0$ eq. piano tang. a Σ in P_0

$\frac{\partial F}{\partial x}(P_0) \neq 0 \Longrightarrow \exists!\, g : B_\delta(2,1) \to B_{\delta'}(0)$ di classe $\mathcal{C}^1$, tale che $g(2,1) = 0$ e

$$F(g(y,z), y, z) = 0$$

La param. di Σ attorno a P_0 data da g esiste ma non si scrive in forma chiusa

Es. 8

$$\Sigma = \{x + y + z^2 = 2\}$$

$P_0 = (1,1,0)$

$$\sigma(u,v) = \begin{pmatrix} 2-u-v^2 \\ u \\ v \end{pmatrix} \qquad \sigma : \mathbb{R}^2 \to \mathbb{R}^3$$

param. globale di Σ come sup. cartesiana

$$\Rightarrow \sigma(1,0) = P_0 = (1,1,0)$$

$$J\sigma(u,v) = \begin{pmatrix} -1 & -2v \\ 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\Rightarrow J\sigma(1,0) = \begin{pmatrix} -1 & 0 \\ 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \text{rango } 2$$

$$\Rightarrow \vec{n}(1,0) = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

$$\Rightarrow 1 \cdot (x-1) + 1 \cdot (y-1) = 0$$

$$x + y = 2$$

piano tang. a Σ in P_0