2. Limiti

Es. 4

$$\lim_{(x,y)\to(0,0)} \frac{x-y}{(x+y)^3} \qquad Dom(f) = \{y \neq -x\}$$

$$f|_{\{x=0\}}(y) = \frac{-y}{y^3} = -\frac{1}{y^2} \longrightarrow -\infty$$

$$f|_{\{y=0\}}(x) = \frac{1}{x^2} \longrightarrow +\infty$$

$\Longrightarrow$ il lim. non esiste

Es. 3

$$\lim_{(x,y)\to(0,0)} \frac{\sin(x^2+2y^2)}{\sqrt{x^2+y^2}} \qquad Dom(f) = \mathbb{R}^2 \setminus \{(0,0)\}$$

$x^2+y^2 \neq 0$

$$\frac{\sin(x^2+2y^2)}{\sqrt{x^2+y^2}} = \underbrace{\frac{\sin(x^2+2y^2)}{x^2+2y^2}}_{\to 1} \cdot \boxed{\frac{x^2+2y^2}{\sqrt{x^2+y^2}}}$$

1 (lim. notevole) $\qquad$ 0 (vedi sotto)

$$g(x,y) := \frac{x^2+2y^2}{\sqrt{x^2+y^2}}$$

$$g\big|_{\{x=0\}}(y) = \frac{2y^2}{\sqrt{y^2}} = \frac{2y^2}{|y|} \overset{y\neq 0}{=} 2|y| \to 0$$

$$g\big|_{\{y=0\}}(x) \overset{x\neq 0}{=} |x| \to 0$$

$$g\big|_{\{y=\lambda x\}}(x) = \frac{x(1+2\lambda^2)}{\sqrt{x^2(1+\lambda^2)}} \overset{x\neq 0}{=} \underbrace{|x|\frac{1+2\lambda^2}{\sqrt{1+\lambda^2}}}_{\to 0} \quad \forall \lambda \in \mathbb{R}$$

$$0 \leq \underbrace{\boxed{\frac{x^2+2y^2}{\sqrt{x^2+y^2}}}}_{\to 0} \leq \frac{2(x^2+y^2)}{\sqrt{x^2+y^2}} = \underbrace{2\sqrt{x^2+y^2}}_{\to 0}$$

per teorema del confronto $\Rightarrow \lim_{(x,y)\to(0,0)} \frac{\sin(x^2+2y^2)}{\sqrt{x^2+y^2}} = 0$

Es. 11

$$\lim_{(x,y)\to(0,0)} \frac{x^3y^3}{x^4+y^4} \qquad Dom(f) = \mathbb{R}^2 \setminus \{(0,0)\}$$

$$0 \leq \left| \frac{x^3y^3}{x^4+y^4} \right| \leq \left| \frac{x^{\cancel{3}}y^{\cancel{3}}}{2\cancel{x^2}\cancel{y^2}} \right| = \frac{1}{2}|xy| \to 0$$

$a^2+b^2 \geq 2ab$
$\forall a, b$

$\Rightarrow x^4+y^4 \geq 2x^2y^2$

Es. 8

$$\lim_{(x,y)\to(0,0)} \frac{x^3y^2}{x^2-y^2} \qquad Dom(f) = \mathbb{R}^2 \setminus \{y=\pm x\}$$

Provate $f|_{\{x=0\}}$ e $f|_{\{y=0\}}$.

$$f|_{\{y=x^\alpha\}}(x) = \frac{x^{3+2\alpha}}{x^2-x^{2\alpha}} =$$

$\alpha>0, \alpha\neq 1$
e $x>0$

$$= \begin{cases} \dfrac{x^{3+2\alpha}}{x^2} & \alpha>1 \\ \dfrac{x^{3+2\alpha}}{x^{2\alpha}} & 0<\alpha<1 \end{cases} \to 0$$

$$f\Big|_{\{y=x+x^\beta\}}(x) = \frac{x^3\left(x^2+2x^{\beta+1}+x^{2\beta}\right)}{\cancel{x^2}-\left(\cancel{x^2}+2x^{\beta+1}+x^{2\beta}\right)} =$$

$$= \frac{x^5+\boxed{o(x^5)}}{-2x^{\beta+1}+\boxed{o(x^{\beta+1})}}$$

$2x^{\beta+4}+x^{2\beta+3} = o(x^5)$ dato che $\beta>1$

$-x^{2\beta} = o(x^{\beta+1})$ poiché $\beta>1$

$$\underset{\beta=4}{=} \frac{x^5+o(x^5)}{-2x^5+o(x^5)} = \frac{\cancel{x^5}\left(1+o(1)\right)}{\cancel{x^5}\left(-2+o(1)\right)} \to -\frac{1}{2}$$

$\Rightarrow$ il lim. non esiste